Limnol. Oceanogr., 48(5), 2003, 2055–2061

The seasonal evolution of wind/internal wave resonance in Lake Kinneret

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Abstract

Data from a thermistor chain and wind sensor collected over an annual stratification cycle in Lake Kinneret (Israel) during 2000 were used to investigate the seasonal evolution of wind/internal wave resonance. Internal wave periods determined from an analytical model were compared with observations, and we show that resonance during 2000 occurred during three distinct times of the year—at the onset of stratification (March), during the heating phase (June), and during the cooling phase (November). In all cases, resonance was between the wind and the dominant radial, azimuthal, and vertical mode-one cyclonic (Kelvin) wave previously observed in Lake Kinneret.

Internal waves are a ubiquitous feature of lakes, existing because of wind forcing acting on a stratified water column, where the stratification is generally due to temperature. Wind applied to the surface can cause a surface setup of water at the downwind end. This pressure force is balanced by a tilting of the metalimnion in the opposite sense—that is, downward at the downwind end (Spigel and Imberger 1980). When the wind forcing relaxes, the water surface oscillates (a barotropic wave), as does the metalimnion (a baroclinic wave). The two motions can effectively be considered to be decoupled (Monismith 1985), because they form orthogonal solutions to the wave equation. It is the internal (baroclinic) waves that are of interest in the present study, because they provide much of the energy and motion for biogeochemical processes in lakes.

In large lakes, the picture becomes complicated, because the effects of the Earth's rotation begin to influence the frequency and structure of the internal waves. Internal gravity waves in large lakes take the form of cyclonic (Kelvin and/or Poincaré) or anticyclonic (Poincaré) waves (Antenucci et al. 2000; Antenucci and Imberger 2001), where the wave crests rotate around the perimeter of the lake basin. Antenucci and Imberger (2001) showed that the frequency, as well as the ratio of potential : kinetic energy, of internal gravity waves in approximately circular or elliptic lakes was dependent on the aspect ratio, radial mode, azimuthal mode, and Burger number. The Burger number is defined as c/Lf, where c is the phase speed of internal waves in a nonrotating system, L is some length scale characterizing the basin width, and f is the Coriolis parameter. For a small Burger number, rotation dominates over gravity, and waves approach the oceanic case (known as inertial oscillations) in which the motion is dominated by the kinetic energy signal. For a large Burger number, gravity dominates over rotation, and waves approach the nonrotating case of equal partitioning between potential and kinetic energy. Using the Burger number classification, Antenucci and Imberger (2001) were able to describe the characteristics of the dominant internal waves in Lake Kinneret, Israel.

Wind forcing of lakes is typically periodic in some sense because of periodicity in weather patterns. The wind over Lake Kinneret has a dominant wind-forcing return period of 24 h (Neumann and Stanhill 1978), Lake Lugano has a period of 60–90 h (Mysak et al. 1985), and Loch Ness has a period of ~50 h (Thorpe 1974). The internal waves in these systems have periods of similar magnitude, which raises the possibility of resonant forcing of the internal waves by the wind. Both Mysak et al. (1985) and Thorpe (1974) discussed the possibility of this resonant interaction between the wind and internal wave field; however, the spectral peaks in the wind field were not sufficiently sharp or the theoretical model not sufficiently complete to conclude that resonance was occurring. Antenucci et al. (2000) presented evidence of the resonant forcing of the dominant vertical, radial, and azimuthal mode-one cyclonic (Kelvin) wave in Lake Kinneret, which during the time of their measurements had a period of ~24 h.

The objective of the present article is to trace the seasonal evolution of wind and internal wave resonance in Lake Kinneret, using the Burger number classification of Antenucci and Imberger (2001). Field data were analyzed using a wavelet transform (Torrence and Compo 1998) to determine the seasonal evolution of the nonrotating phase speed, c, and hence the evolution of the Burger number. This was used to determine the natural frequencies of the lake and to show how these frequencies coincide with forcing frequencies at several times during the year. The internal wave response was determined via wavelet transforms of thermistor chain data and are presented both as an amplitude and energy response. The implications of wind/internal wave resonance for water quality are also discussed.

Methods

The site for the field investigation was Lake Kinneret, Israel (Fig. 1). The lake is ~22 × 15 km and is oriented

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Acknowledgments

We acknowledge the field support and technical staff from the Kinneret Limnological Laboratory (KLL) for providing assistance, particularly Tzahi Rosenberg for maintaining the lake station and helping in the transfer of data from KLL to the Centre for Water Research, and Peter Yeates, who provided the time series of the lake number.

The field component of this project was funded by the Israeli Water Commission.

This is Centre for Water Research reference ED-1276-JA.

north-south, with a maximum depth of 42 m and a surface area of 167 km^2. During the months of April–October, a daily westerly sea breeze blows over the lake that regularly reaches speeds of 15 m s^{-1} at 10 m above the water surface. The water column remains stratified from late February through late December.

A thermistor chain that consists of an array of 20 thermistors measuring to an accuracy of 0.01°C was fitted with a wind sensor located 1.5 m above the water surface and placed at Sta. A (Fig. 1). During the weakly stratified periods, the location of the station was not ideal for the investigation of Kelvin waves; however, the positioning was suitable once stratification increased. Resolution was 1 m in the metalimnion and up to 4 m in the weakly stratified hypolimnion. Data were collected at 10-min intervals and were telemetered to a shore station. The station was installed in early January 2000 and continues to operate. Here we consider only data collected between January and December 2000.

Theoretical internal wave characteristics

The evolution of the background stratification in Lake Kinneret during 2000 is shown in Fig. 2a. This was determined by finding the average isotherm depth (Thorpe et al. 1996) over 2-d intervals. The lake shows the typical seasonal stratification pattern of a monomictic lake, with an approximately constant rate of heating until day 150, a long period of strong stratification (until day 250), and epilimnion deepening as the lake begins to cool. To determine the evolution of phase speed, c, we solved the eigenvalue problem for long waves (LeBlond and Mysak 1978)

$$\frac{d^2\Psi(z)}{dz^2} + \frac{N^2(z)}{c^2}\Psi(z) = 0 \qquad (1)$$

where $\Psi(z)$ is the vertical velocity eigenfunction and $N(z)$ is the buoyancy frequency profile, computed from

$$N^2(z) = -\frac{g}{\rho_0}\frac{d\rho(z)}{dz} \qquad (2)$$

where g is the gravitational constant, ρ_0 is a reference density, and $\rho(z)$ is determined from the background temperature profiles in Fig. 2a. It was assumed that salinity had a negligible effect on the density gradient. The eigenvalues c were found for vertical mode-one, -two, and -three waves, because these waves have previously been observed in Lake Kinneret (Antenucci et al. 2000).

The Burger number as a function of time was thus computed as

$$S_i(t) = c_i(t)/Lf \qquad (3)$$

where i indicates the vertical mode. The constant values for L and f ($L = 7{,}500$ m and $f = 7.81 \times 10^{-5}$ rad s^{-1} for Lake Kinneret, an inertial period of 22.3 h) results in the seasonal evolution of the nonrotating phase speed c_i; the Burger number S_i follows the same pattern (Fig. 2b). During the unstratified period, the Burger number was effectively 0. As the lake heated during spring, S_i gradually increased, reaching a maximum sometime in autumn, before the lake began to cool until again becoming unstratified. The vertical mode-one Burger number S_1 reached a maximum of 0.8, whereas the higher vertical-mode Burger numbers reached maxima of <0.3. This implies that rotation will always affect the higher vertical-mode waves to a greater extent than the vertical mode-one waves.

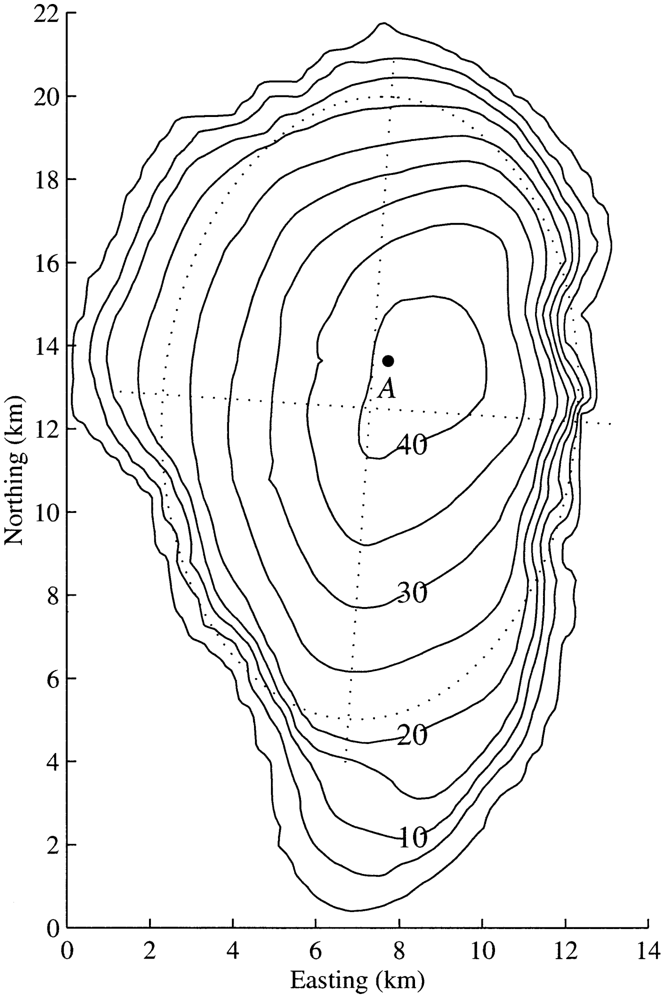

Fig. 1. Lake Kinneret bathymetry with the location of the thermistor chain and wind sensor during 2000. An ellipse of aspect ratio 2 : 3 (10,000 : 15,000 m) is superimposed for use in the discussion. The origin of the map grid is situated at 35.51°N, 32.70°E.

The dependence of the frequency and ratio of potential to kinetic energy on the Burger number was discussed by Antenucci and Imberger (2001), although we include a short discussion here, for completeness. In lakes where the Earth's rotation plays a role in the basin-scale internal wave dynamics, there are two main types of gravity waves: cyclonic (those whose phase progresses in the same sense as the rotation) and anticyclonic. The cyclonic waves (Kelvin waves for $\omega < f$, otherwise Poincaré waves) can exist at both super- and subinertial frequencies. The anticyclonic waves (Poincaré waves for all ω) exist only at superinertial frequencies ($\omega < f$). Cyclonic waves have a potential : kinetic energy ratio that is close to unity, whereas anticyclonic waves have a ratio that ranges from 0 to unity as the Burger number increases. For a low Burger number, the anticyclonic waves

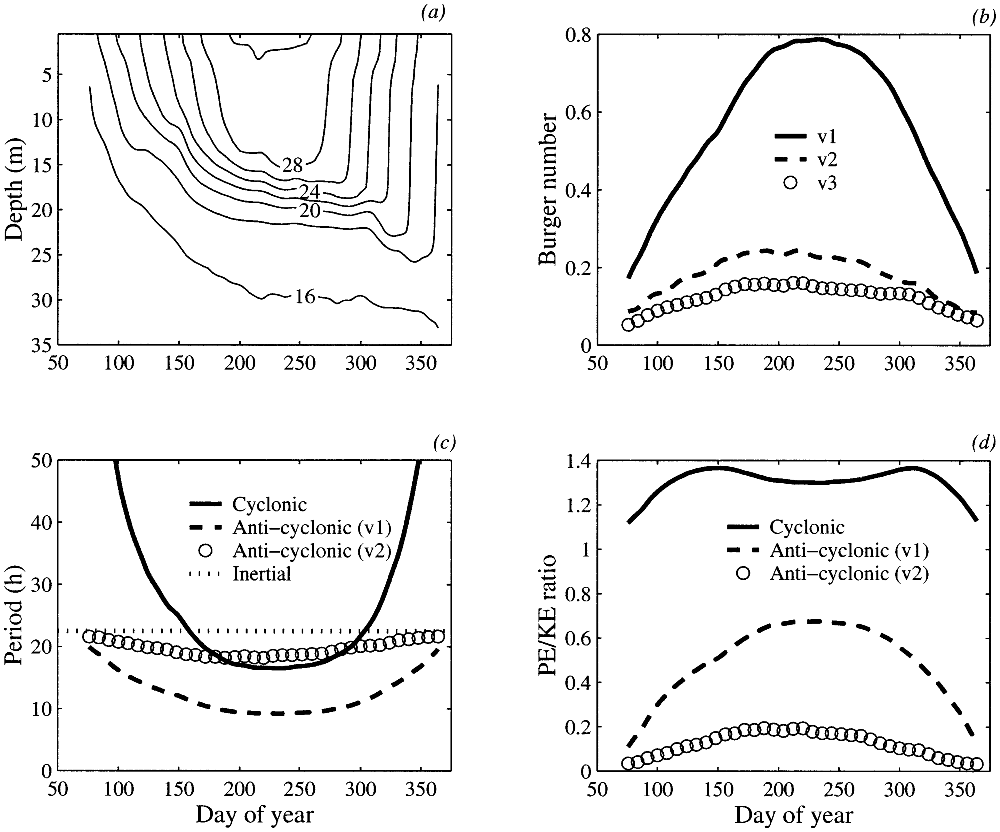

Fig. 2. (a) Evolution of the background stratification during 2000, when the background stratification was determined from the average isotherm position over 2-d intervals. (b) Evolution of the Burger number for the first three vertical modes. (c) Evolution of the natural periods of the dominant basin-scale internal waves in Lake Kinneret. (d) Evolution of the potential : kinetic energy ratios.

are dominated by rotation, and the kinetic energy signal is large. For a high Burger number, the restoring force of gravity dominates, and both the cyclonic and anticyclonic waves approach gravity waves in nonrotating systems, where the potential : kinetic energy ratio is exactly unity for small-amplitude waves (Gill 1982).

We used the results of Antenucci and Imberger (2001) to determine the natural frequencies of the fundamental basin-scale modes in Lake Kinneret, found by Antenucci et al. (2000) to be a vertical mode-one cyclonic wave of the first radial and azimuthal mode and vertical mode-one, -two, and -three anticyclonic waves of the first radial and azimuthal mode. We assumed the lake to be an ellipse of aspect ratio 2 : 3, with a major axis length of 15,000 m, according to Antenucci and Imberger (2001). The seasonal evolution of the theoretical periods of the vertical mode-one and -two waves are presented in Fig. 2c, along with the evolution of the potential : kinetic energy ratios (Fig. 2d).

Because the Burger number increases with increasing stratification, the period of the dominant internal wave in Lake Kinneret, the vertical mode-one, fundamental cyclonic wave, decreases from essentially infinite to a minimum of 18 h at the peak stratification (mid-August, day 230). Because the Burger number represents a balance of the forces of gravity and rotation, an increasing Burger number indicates an increasing trend for gravity to dominate the internal wave dynamics, and the effect of rotation is constrained by the lake size such that waves are reflected faster than they can turn. During the autumn cooling phase, as the stratification and Burger number decrease, the system returns to one dominated by rotation.

The period of the anticyclonic waves has an upper limit determined by the inertial period (22.3 h in Lake Kinneret), with the vertical mode-one wave having a minimum period of 10 h and the vertical mode-two wave having a minimum period of 19 h at the peak stratification. As with the cyclonic case, these waves see large changes in the relative dominance of gravity and rotation as the Burger number varies. This is particularly evident when considering the potential : kinetic energy ratios, which range between 0 and 0.65 for the vertical mode-one wave and between 0 and 0.2 for the vertical mode-two wave. A ratio of 0 is indicative of the dominance of rotation over gravity, with the ratio increasing to unity as the importance of gravity increases. It is clear that the importance of rotation and gravity will vary during the year when considering the dynamics of the vertical mode-one wave, whereas rotation will always be important for the dynamics of the vertical mode-two wave. The anticyclonic waves, if present, are therefore more likely to be observed through the kinetic energy signal (i.e., by velocity measurements) than with potential energy measurements such as temperature fluctuations

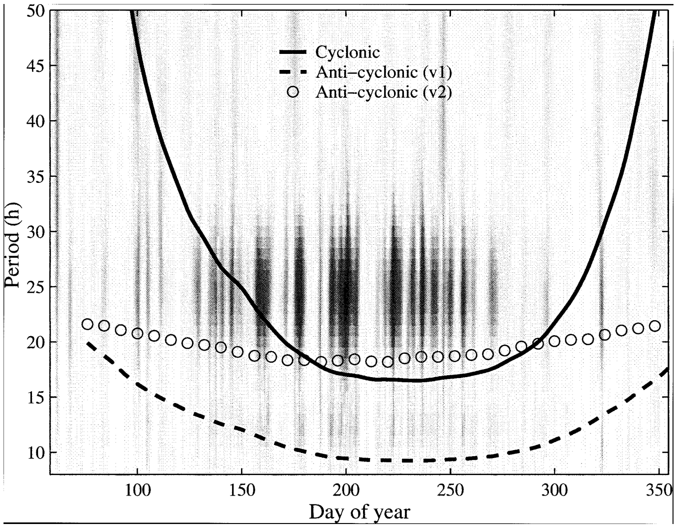

Fig. 3. Continuous wavelet transform of the square of the wind speed, overlaid with the natural periods of the dominant basin-scale internal waves.

or isotherm displacements at all times of the year but particularly during the period of weaker stratification.

Forcing characteristics

Although the natural frequencies and characteristics of the waves may change with the seasons, it is possible that no waves would ever be observed if the system were not sufficiently forced. The strength and frequency content of the wind forcing over Lake Kinneret during 2000 is shown in Fig. 3, where we present the continuous wavelet transform of the square of the wind speed. Note that a Morlet mother wavelet was used (Antenucci et al. 2000) in which there was high temporal resolution and low-frequency resolution. The wind field in Lake Kinneret showed a strong 24-h component from early May through early October (days 130–260), with a weaker signal also present at 12 h from June to August (days 180–240). This 24-h component was due to a daily westerly sea breeze that occurs from spring through the autumn months (Serruya 1974). Superimposed on Fig. 3 are the theoretical internal wave periods presented in Fig. 2c. There are several periods in which the periodicity in the wind field matched the theoretical internal wave periods, raising the possibility of resonant interaction of the wind and internal waves in Lake Kinneret during 2000. This is particularly evident around days 150 and 300 at ~25 h and on day 100 at ~48 h. Resonance between the wind and internal waves in Lake Kinneret during 1998 was discussed by Antenucci et al. (2000), although not in a seasonal context.

Internal wave response

The seasonal evolution of the internal wave response was determined by converting the temperature signal to a potential energy signal by integrating over the depth (Antenucci et al. 2000). A continuous wavelet transform was applied to this signal, to determine the strength of the internal wave field as a function of frequency and time. A two-layer approximation was then used for each 2-d period, to determine the equivalent background stratification. The amplitude of the perturbation to this two-layer approximation required to match the magnitude of the continuous wavelet transform of the measured potential energy signal was then determined (Antenucci et al. 2000). The variation of this amplitude with frequency and time is presented in Fig. 4b, with the theoretical periods of the dominant vertical mode-one waves overlaid. To give an indication of the energy response, as opposed to just the amplitude response, the variation of the perturbation potential energy (Gill 1982) $0.5\Delta\rho g\Delta z^2$ is presented in Fig. 4c, where $\Delta\rho$ represents the density difference between the upper and lower layers and Δz is the displacement plotted in Fig 4b. The seasonal variation of the Lake number L_N (Imberger and Patterson 1990), a parameter that compares the stability of the stratification with the disturbing force due to the wind, is presented in Fig. 4a, where small Lake numbers are indicative of upwelling events and strong initial thermocline tilts. Also included is the interface deflection d (Antenucci et al. 2000), defined as

$$d = \left(1 - 2\frac{x}{L}\right)\frac{h_2}{h_1 + h_2}\frac{L}{2Ri} \tag{4}$$

where h_1 and h_2 represent the thickness of the upper and lower layers respectively, L is the basin dimension along the major wind axis (12,000 m), x is the station location relative to the upwind end (5,000 m), and Ri is the bulk Richardson number c^2/u_*^2, where u_* was taken as the mean daily wind-induced shear velocity. The interface deflection d thus represents the initial disturbance of the interface at Sta. A due to the wind, based on a two-layer stratification.

Up until day 90, there were strong internal wave signals at 24 and 12 h, although these did not correspond with any theoretical internal wave period (Fig. 4b). These motions can be attributed to the low lake number, which resulted in strong forced motions. From an energy perspective (Fig. 4c), these motions are relatively weak because of the weak stratification. From Fig. 4b, the largest amplitude internal waves observed in Lake Kinneret occurred around day 100, with a period of ~50 h. These waves also had a significant energy signal (Fig. 4c). The 50-h period corresponded with the natural period of the cyclonic vertical mode-one wave at the time. Also occurring at this time, along with an elevated forcing frequency at 24 h, was a strong 50-h periodicity to the wind forcing (Fig. 3) due to the daily sea breeze being weaker on day 99. The wind and isotherm response are shown in Fig. 5a. The conditions around day 100 showed violent oscillations of the metalimnion, with amplitudes up to 5 m close to the node of the oscillation (Fig. 1). We concluded that the internal wave response around day 100 was due to a vertical mode-one cyclonic wave that was forced through resonant interaction with the 50-h component of the wind.

After day 150, there were two features of interest. The first, around day 160, was elevated internal wave activity (amplitude and energy) at the time the period of the vertical mode-one cyclonic wave matched the dominant forcing period of 24 h (Fig. 4b,c). The second was when the natural

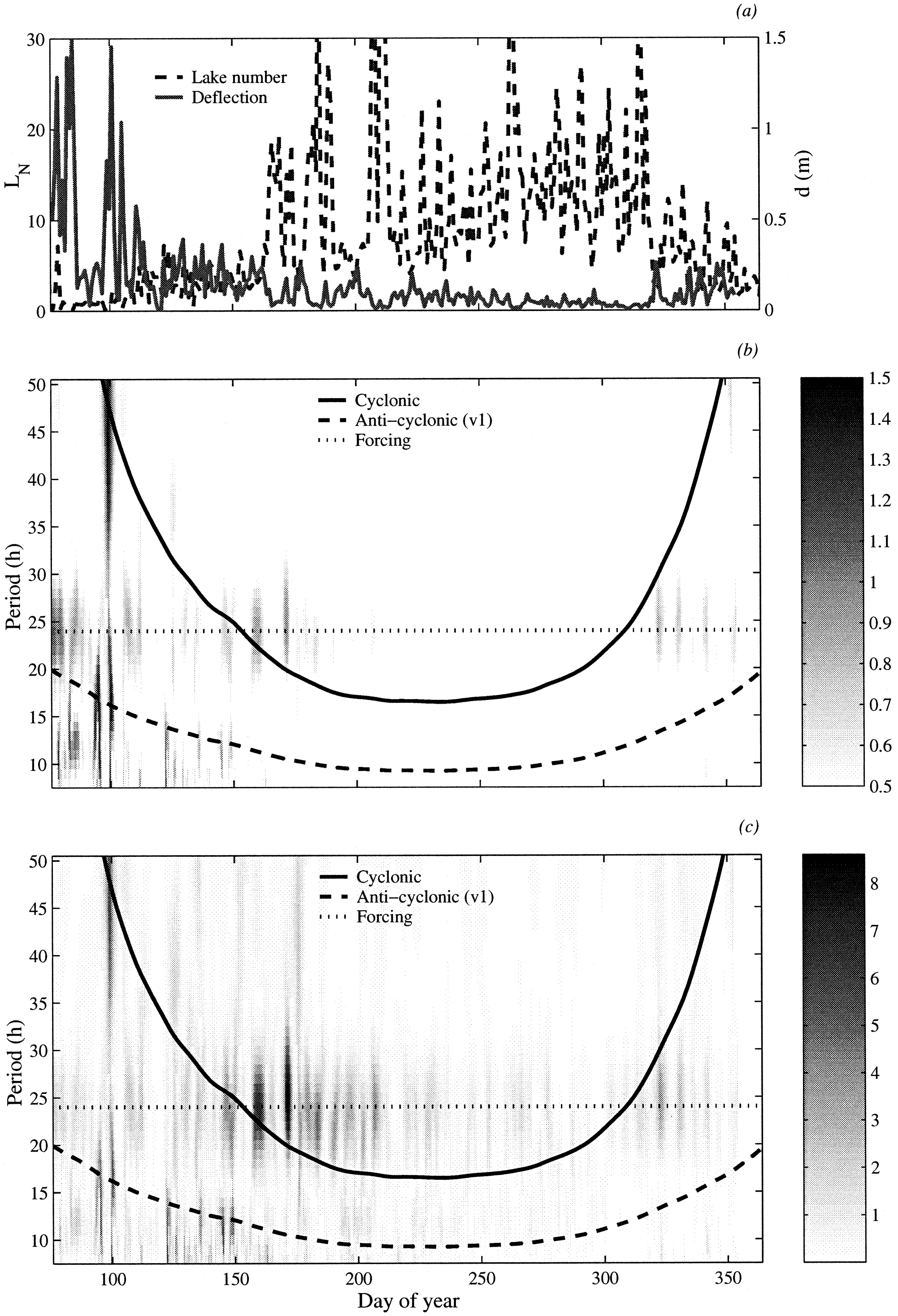

Fig. 4. Evolution of (a) the Lake Number L_N and the initial setup *d* (in m); (b) the equivalent amplitude (in m) of perturbations of a two-layer stratification (log scale); and (c) the perturbation potential energy (in J m^{-2}) of the two-layer stratification. Panels b and c are overlaid with the natural periods of the vertical mode-one cyclonic and anticyclonic waves.

period of the dominant cyclonic wave again equaled ~24 h, around day 320, when elevated levels of internal wave activity were recorded. From Fig. 3, there was an elevated level of wind energy at 24 h. Given that the natural and forcing frequencies during this time coincided, we concluded that resonance between the wind and dominant internal wave was responsible for the elevated energy in the internal wave field at these times.

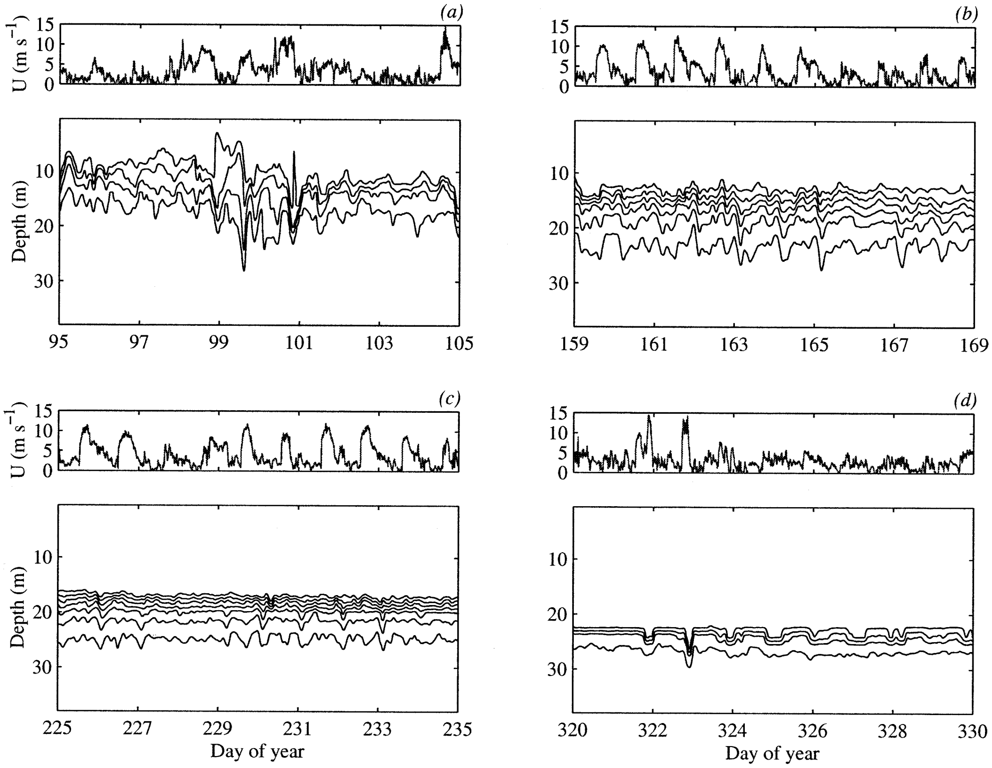

Fig. 5. Isotherms and wind speeds during (a, b, and d) resonant and (c) nonresonant periods. Contour interval is 0.75°C for panel a and 1.5°C for panels b–d. The bottom contour is 15.75°C for panel a and 16.5°C for panels b–d. Isotherms in the surface mixed layer have been masked, to emphasize the metalimnion motion.

The importance of resonance in increasing the internal wave amplitudes in the lake was further examined by looking at nonresonant periods. Days 200 and 225 saw low lake numbers similar to those around day 150 but resulted in far lower energy in the internal wave field (Fig. 4b,c). This was apparently due to there being no natural period of oscillation similar to the forcing period at this time.

The wind and isotherm displacements during the resonant and nonresonant periods are presented in Fig. 5. The strong 24-h periodicity in the winds between days 159 and 165 is evident in Fig. 5b. Associated with this was a corresponding 24-h vertical mode-one motion of the metalimnion of amplitude 2–3 m. Between days 225 and 235, a similar period of strong winds resulted in a smaller isotherm response (Fig. 5c) and was attributed to the natural period of the cyclonic wave being ~18 h and so experiencing nonresonant conditions with the wind forcing. On days 321 and 322, two strong wind events occurred 24 h apart, at the same time that the natural period of the cyclonic wave was once again ~24 h. The internal wave generated persisted for 5 d and was by far the largest amplitude oscillation of the metalimnion in the last 5 months of the year (Figs. 4, 5d).

Implications of resonance

What are the implications for these periods of wind/internal wave resonance? The result of resonance is larger amplitude internal waves, in particular the cyclonic, vertical mode-one wave. In Lake Kinneret, the effect of this wave is primarily in the littoral regions on the western shore, where the sloping sides of the lake (~1 : 200) result in a wave of 10 m amplitude traveling ~2 km along the bottom slope in a 12-h period from trough to crest, at a velocity of ~5 cm s^{-1}. Associated with the motion of this wave is the development of a benthic boundary layer, in which the turbidity increases from the deepest to the shallowest stations, in response to resuspension by the swashing action of the internal wave (Shteinman et al. 1997; Nishri et al. 2000). The increased internal wave energy that results during resonant periods is therefore likely to cause increased rates of resuspension, although field experiments would need to be carried out to confirm this.

An analysis of thermistor chain and wind data collected over an annual stratification cycle in the periodically wind-forced Lake Kinneret revealed several periods of strong internal wave activity. It was shown that the largest internal wave energy response occurred when the natural internal wave frequency was similar to the forcing frequency of the wind. This was shown to occur at two different frequencies (24 and 50 h), at three separate times of the year. It is clear from the analysis that resonant forcing of internal waves is a common phenomenon in Lake Kinneret and has implications for transport and mixing, in particular particle resuspension, because of the amplitudes of the waves involved.

References

ANTENUCCI, J. P., AND J. IMBERGER. 2001. Energetics of long internal gravity waves in large lakes. Limnol. Oceanogr. **46:** 1760–1773.

———, ———, AND A. SAGGIO. 2000. Seasonal evolution of the basin-scale internal wave field in a large stratified lake. Limnol. Oceanogr. **45:** 1621–1638.

GILL, A. E. 1982. Atmosphere-ocean dynamics. Academic.

IMBERGER, J., AND J. C. PATTERSON. 1990. Physical limnology. Adv. Appl. Mech. **27:** 303–475.

LEBLOND, P. H., AND L. A. MYSAK. 1978. Waves in the ocean. Elsevier.

MONISMITH, S. G. 1985. Wind-forced motions in stratified lakes and their effect on mixed-layer shear. Limnol. Oceanogr. **30:** 771–783.

MYSAK, L. A., G. SALVADÉ, K. HUTTER, AND T. SCHEIWILLER. 1985. Topographic waves in a stratified elliptical basin, with application to the Lake of Lugano. Phil. Trans. R. Soc. Lond. A **316:** 1–55.

NEUMANN, J., AND G. STANHILL. 1978. The general meteorological background, p. 49–58. *In* S. Serruya [ed.], Lake Kinneret. Dr. W. Junk.

NISHRI, A., J. IMBERGER, W. ECKERT, I. OSTROVSKY, AND Y. GEIFMAN. 2000. The physical regime and the respective biogeochemical processes in the lower water mass of Lake Kinneret. Limnol. Oceanogr. **45:** 972–981.

SERRUYA, S. 1974. Wind, water temperature and motions in Lake Kinneret: General pattern. Verh. Int. Ver. Limnol. **19:** 73–87.

SHTEINMAN, B., W. ECKERT, S. KAGANOWSKY, AND T. ZOHARY. 1997. Seiche-induced resuspension in Lake Kinneret: A fluorescent tracer experiment. Water Air Soil Pollut. **99:** 123–131.

SPIGEL, R. H., AND J. IMBERGER. 1980. The classification of mixed-layer dynamics in lakes of small to medium size. J. Phys. Oceanogr. **10:** 1104–1121.

THORPE, S. A. 1974. Near-resonant forcing in a shallow two-layer fluid: A model for the internal surge in Loch Ness? J. Fluid Mech. **63:** 509–527.

———, J. M. KEEN, R. JIANG, AND U. LEMMIN. 1996. High-frequency internal waves in Lake Geneva. Phil. Trans. R. Soc. Lond. A **354:** 237–257.

TORRENCE, C., AND G. P. COMPO. 1998. A practical guide to wavelet analysis. Bull. Am. Meteorol. Soc. **79:** 61–78.

Received: 25 April 2002
Amended: 28 March 2003
Accepted: 3 April 2003